15 **Mathematics** Set-II/12

Time : 3 Hrs. **Max. Marks : 100**

General Instructions:

1. All questions are compulsory
2. The question paper consists of 26 questions and is divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section comprises of 13 questions of four marks each and section C comprises of 7 questions of six marks each.
3. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions. Graph is required for Q. No. 23 (b)

Write your email id.....

SECTION - A

1. Let A = {1, 2, 3, 4 ..... 15, 16} and let R be a relation in A, given that R = {a, b): b = a^2}. Find R .

2. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ then find $\cos^{-1}x + \cos^{-1}y$.

3. If matrix A = diag [-1 4 2] then find 2A + 4I.

4. Find the slope of normal to the curve $x = \frac{1}{t}, y = 2t$ at $t = 2$.

5. Evaluate $\int x^x(1+\log x)dx$.

6. Prove that $(\vec{a}.\vec{b})^2 \leq |\vec{a}|^2|\vec{b}|^2$.

SECTION - B

7. (a) If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $fof(x) = x$ for all $x \neq \frac{2}{3}$. Also, find the Inverse of f.

   (b) If * : R × R → R defined as a*b = a Show that * is not commutative but Associative.

8. Prove that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$.

9. Use Product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:

$x - y + 2z = 1$
$2y - 3z = 1$
$3x - 2y + 4z = 2$

10. Using Properties of determinants, Prove that

$$\begin{vmatrix} a & a-b & b+c \\ b & b-c & c+a \\ c & c-a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

11. If $\begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$I + A = (I - A) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

12. Discuss the continutity of the function at $x = 0$,

$$f(x) = \begin{cases} \dfrac{27^x - 9^x - 3^x + 1}{1 - \cos x} & ; \text{if } x \neq 0 \\ 4(\log 3)^2 & ; \text{if } x = 0 \end{cases}$$

OR

Find the value of k for which $f(x) = \begin{cases} \dfrac{\log(1+ax) - \log(1-bx)}{x} & ; \text{if } x \neq 0 \\ k & ; \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.

13. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x - y)$ then prove that $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$

OR

If $y = \sqrt{\sin x + \left(\sqrt{\sin x + \ldots\ldots\infty}\right)}$ then find $\dfrac{dy}{dx}$

14. Evaluate: $\int \dfrac{\text{Sin}(x-\alpha)}{\text{Sin}(x+\alpha)}\, dx$

OR

Evaluate: $\int Sec^3 x\, dx$

15. Evaluate $\int \dfrac{1}{x(x^6+1)}\, dx$

M[2]

16. For the given differential equation, find the particular solution

$(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}; y=0$ when $x=1$

17. Solve the Differential Equation, $\sin^{-1}\left[\frac{dy}{dx}\right]=x+y$.

OR

$x\frac{dy}{dx}=y(\log y-\log x+1)$

18. Find the vector equation of the plane passing through three points with position vectors $\hat{i}+\hat{j}-2\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$.

19. In a backward state, there are 729 families having six children each. If probability of survival of a girl is $\frac{1}{3}$ and that of boy is $\frac{2}{3}$, find the number of families having 2 girls and 4 boys. Do you believe that a female child is neglected in backward areas? What steps will you take to restore respect of a female child in society?

SECTION-C

20. A square tank of capacity 250 m^3 has to be dug out. The cost of the land is ₹ 50 per Sq. m. The cost of the digging increases with the depth and for the whole tank, it is ₹ $(400xh^2)$, where h metres is the depth of the tank, what should be the dimensions of the tank so that the cost is minimum?

OR

Find the points on the curve $y^3+3x^2=12y$ where the tangent is vertical.

21. Evaluate $\int_0^{2\pi}\frac{1}{(1+e^{\sin x})}dx$

22. Find the area bounded by the curves $\left\{(x,y):|x-1|\le y\le\sqrt{5-x^2}\right\}$, by using integration.

23. a) A fruit grower has two godowns A and B to store the fruits of capacity 100 and 50 quintals respectively. From these places, the fruits to be supplied to three Fruit Merchants P, Q, R. The monthly requirements of these merchants are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the fruit merchants are given below.

Transportation Cost per quintal (in Rs.)

From/To	A	B
P	6	4
Q	3	2
R	2.50	3

Formulate this as a linear programming problem for the minimum cost of transportation from godowns to fruit merchants

(b) Solve the given linear programming Problem graphically

Maximise $= 3x + 9y$

Subject to the constraints: $x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x, y \geq 0$

24. Find the length and the equation of the line of shortest distance between the lines:

$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} + \frac{z-6}{4}$

25. (a) If the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar then prove that vectors $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are also coplanar.

(b) If $\vec{a}$, $\vec{b}$, $\vec{c}$, be three vectors such that $|\vec{a}|=5$, $|\vec{b}|=12$, $|\vec{c}|=13$ and $\vec{a}+\vec{b}+\vec{c}=0$ then find $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.

26. In answering a question of a MCQ test with 4 choices per question, a student knows the answer, guesses or copies the answer. Let $\frac{1}{2}$ be the probability that he knows the answer, $\frac{1}{4}$ be the probability of guessing and $\frac{1}{4}$ that he copies it. Assuming that a student, who copies the answer, will be correct with the probability $\frac{3}{4}$ what is the probability that he knows the answer, given that he answered it correctly.

OR

Find the mean, variance and standard deviation of the number obtained on a throw of an unbiased dice.